Portfolio Omega and Optimization

Omega is a performance measure that takes all information about a return distribution of an asset into account. It is used in many areas, but rarely in a portfolio optimization because of the complexity in such computations. In this paper, we propose a new measure, called portfolio Omega, which can be easily used in a portfolio optimization. Portfolio Omega possesses many desired characteristics: It can be decomposed according to investment strategies or a sector classification; it is an extension of Omega (Keating and Shadwick, 2002b) and the Sharpe-Omega (Kazemi et. al, 2004); and portfolio Omega allows investors to put their views into an optimization, and to set their funding cost and investment strategies in the measure. Also, the paper will provide inside information about the new measure, diversification, loss threshold, and downside risk of a portfolio through an empirical work. The empirical work reveals portfolio Omega is consistent with practices used in the industry.

INTRODUCTION

Keating and Shadwick (2002b) introduced a performance measure called Omega. Omega is the ratio of probability-weighted gains and losses of an asset for a given loss threshold. Omega requires no assumption on the return distribution of the asset, and takes all information from the distribution into account (it does require the distribution to be specified or estimated). By contrast, the Sharpe ratio (calculated by subtracting the risk-free rate from the expected return of an asset and dividing the result by the standard deviation of the return distribution) involves only the first two moments of the return distribution of the asset (Sharpe, 1966), ignoring the information from the third or higher moments of the distribution.

Omega can be directly interpreted as the ratio of the price of a European call option to a European put option written on the asset (Kazemi et. al, 2004). The strike price of the options is a loss threshold. Based on this interpretation, the probability-weighted gain in Omega is the “cost” of acquiring the return above the threshold, while the probability-weighted loss is the “cost” of protecting the return below the threshold. Furthermore, Omega can be computed by the Black-Scholes formula if the asset price is log-normally distributed or by some numerical method if the return is not log-normally distributed.

Investors can use Omega for many applications including analyzing asset performance, evaluating mutual funds, and ranking hedge funds. Determining an ex-post value of Omega does not usually present any problem; the difficulty lies in applying Omega to a portfolio optimization. Since Omega takes all information from a return distribution, which closely depends on the weights of assets in a portfolio optimization, the information changes as the weights are altered. Whenever investors face a challenge to handle changes in information, they have a difficulty to find optimal weights in a portfolio.

In this paper, we will derive a new measure using a replicated portfolio. The new measure not only helps investors overcome the difficulty in a portfolio optimization, but also makes it easy to form and solve an optimization problem. The new measure possesses superb characteristics. It is
easily decomposed according to investment strategies or a sector classification; it is an extension of Omega (Keating and Shadwick, 2002b) and the Sharpe-Omega (Kazemi et al., 2004); and it measure allows investors to put their views into an optimization, and to set their funding cost and investment strategies in the measure. Also, we will provide inside information about the new measure, diversification, loss threshold, and downside risk of a portfolio through an empirical work. The empirical work will demonstrate portfolio Omega is consistent with practices used in the industry. We will call the new measure as portfolio Omega in the rest of the paper.

We organize the paper into five sections. In the second section, we recall the interpretation of Omega given by Kazemi et al. (2004). Then we derive portfolio Omega using a replicated portfolio. In the third section, we show and discuss characteristics and a drawback of portfolio Omega. We provide possible solutions to deal with the drawback. In the fourth section, we analyze the portfolio Omega through an empirical work. The analyses illustrate the relationships among portfolio Omega, diversification, loss threshold, and downside risk of the portfolio. We conclude in the last section.

**DERIVATION**

In this section, we will briefly recall Omega defined by Keating and Shadwick (2002b), the interpretation of Omega given by Kazemi et al. (2004), and further discuss the difficulty of using Omega in a portfolio optimization. We will derive portfolio Omega using a replicated portfolio and show an equivalent measure at the end of section.

Let $F(x) = \Pr\{X \leq x\}$ be the cumulative distribution of one-period return of an asset $x$ defined on the interval $(a, b)$ and $L$ be a loss threshold for the asset. Keating and Shadwick (2002b) define Omega as

$$
\Omega(L) = \frac{\int_{L}^{b}(1 - F(x))dx}{\int_{a}^{L}F(x)dx},
$$

(1)

where $\int_{L}^{b}(1 - F(x))dx$ and $\int_{a}^{L}F(x)dx$ are respectively the probability-weighted gain and loss of the asset with respect to the threshold $L$.

Omega has many desirable features as a performance measure. One is that it can be interpreted as the ratio of a European call option and a European put option written on the asset (Kazemi et al., 2004). Under the interpretation, the Omega is the ratio:

$$
\Omega(L) = \frac{C(L)}{P(L)},
$$

(2)

---

1 The interpretation requires that the initial price of the asset is 1 since the variable $X$ represents a return of the asset.
where \( C(L) \) and \( P(L) \) correspond to the prices of the call option and put option\(^2\). Both options have the loss threshold \( L \) as their strike price.

It is difficult to use Omega in a portfolio optimization. The difficulty comes from the fact that the call and put options in Omega are written on the portfolio. The prices of options are affected by the return distribution of each asset in the portfolio, the correlation among these assets, and weights of the assets in the portfolio. Since the options’ prices are not linear (or quadratic) functions in the weights, a linear (or quadratic) programming may not be used to find optimal weights, and the optimization becomes increasingly computationally demanding as the number of assets in the portfolio increases.

In order to overcome the difficulty of Omega in a portfolio optimization, we need to look for a new measure similar to Omega, but the measure does not have the difficulty in an optimization. The derivation of a new measure consist of three steps. We first begin with working on a return replication of asset \( x \). Since

\[
x - L = \max(x - L, 0) - \max(L - x, 0),
\]

It follows that

\[
E(x) - L = E[\max(x - L, 0)] - E[\max(L - x, 0)],
\]

where \( E \) is the expectation operator under the return distribution of the asset. If we denote

\[
C(x, L) = e^{-r_f} E[\max(x - L, 0)]
\]

and

\[
P(x, L) = e^{-r_f} E[\max(L - x, 0)],
\]

\( r_f \) is the continuous compounding risk-free rate over the period, we have

\[
E[x] - L = e^{r_f} C(x, L) - e^{r_f} P(x, L).
\] (3)

The equation in (3) is put-call parity as \( C(x, L) \) and \( P(x, L) \) correspond to prices of a European call and a European put with the same striking price \( L \) in a risk neutral world\(^3\). The put-call parity suggests that the expected return of the asset over the strike price is the price differential between the call option and put option at the end of the period. It implies that the asset can be

\(^2\)The prices are under the true measure, instead of the risk neutral measure.

\(^3\) In a risk neutral world, an option price is the expected value of future payoff of the option discounted at the risk-free rate. The risk neutral world is unnecessary if the expected value is discounted by a rate embedded with a risk premium.
replicated with a long call, a short put, and cash before applying the continuous compounding rate to the call and put prices\(^4\).

Next, we apply put-call parity to a portfolio. Suppose that a portfolio consists of \( n \) assets \( x_1, x_2, \ldots, x_n \). The \( i^{th} \) asset \( x_i \) has a weight \( w_i \) at the beginning of the period. By (3), the put-call parity for the asset \( x_i \) is

\[
E[x_i] - L_i = e^{r_i} C(x_i, L_i) - e^{r_i} P(x_i, L_i).
\]  

(4)

Multiplying both sides of the \( i^{th} \) equation in (4) by the weight \( w_i \) and taking a summation of the \( n \) resulting equations, we have

\[
\sum_{i=1}^{n} w_i E[x_i] - \sum_{i=1}^{n} w_i L_i = e^{r_i} \sum_{i=1}^{n} w_i C(x_i, L_i) - e^{r_i} \sum_{i=1}^{n} w_i P(x_i, L_i)
\]

If the random variable \( x_p \) represents the portfolio’s return: \( \sum_{i=1}^{n} w_i x_i \) and the portfolio loss threshold \( L_p \) stands for the weighted average loss threshold: \( \sum_{i=1}^{n} w_i L_i \), the above formula becomes put-call parity of the portfolio:

\[
E[x_p] - L_p = e^{r_p} \sum_{i=1}^{n} w_i C(x_i, L_i) - e^{r_p} \sum_{i=1}^{n} w_i P(x_i, L_i).
\]  

(5)

The equation in (5) tells us that the portfolio can be replicated by a group of call options, a group of put options, and cash. If the portfolio contains a long position in asset \( x_i \) with weight \( w_i \), it can be treated as holding a long call \( C(x_i, L_i) \), a short put \( P(x_i, L_i) \), and long cash \( e^{-r_i} L_i \), each of which has \( w_i \) units. In case of a negative weight \( w_i \), the portfolio can be viewed as comprising a short call \( C(x_i, L_i) \), a long put \( P(x_i, L_i) \), and short cash \( e^{-r_i} L_i \), where each of the call, put, and cash has the same unit \(- w_i\). Hence, the portfolio can be treated as holdings of cash, the call options, and the put options.

To simplify our later discussion, we will assume that the portfolio owns a long position of each asset. Under this assumption, the replicated portfolio will long all call options, short all put options, and holds positive cash.

If an investor carries a portfolio and wishes to gain his wealth, he certainly desires to increase the portfolio value. On the other hand, he may want to avoid a loss at a given threshold in the portfolio. In this case, the investor likes to have a high weighted average price of all call options and a low weighted average price of all put options for the given loss threshold. Put another way, he enjoys a higher ratio of the weighted average price of call options to the weighted average price of put options.

\(^4\) We label \( L \) as a loss threshold in the definition of Omega while calling \( L \) as a strike price with respect to the options \( C(x, L) \) and \( P(x, L) \). We also call \( L \) as cash in a replication of an asset or portfolio. We will use these terms interchangeably in the rest of paper.
price of put options. As a result, the investor likes to choose the ratio as a performance measure for the portfolio. From (5), the ratio is

\[
\Omega_p(w_1, w_2, \ldots, w_n) = \frac{\sum_{i=1}^{n} w_i C(x_i, L_i)}{\sum_{i=1}^{n} w_i P(x_i, L_i)},
\]

(6)

which is called portfolio Omega. The portfolio Omega in (6) is expressed as a function in weights of assets in the portfolio.

The portfolio Omega can be scrutinized through the transformation:

\[
\begin{align*}
\Omega_p(w_1, w_2, \ldots, w_n) &= \frac{\sum_{i=1}^{n} w_i C(x_i, L_i)}{\sum_{i=1}^{n} w_i P(x_i, L_i)} \\
&= \frac{e^{r} \sum_{i=1}^{n} w_i C(x_i, L_i) - e^{r} \sum_{i=1}^{n} w_i P(x_i, L_i)}{e^{r} \sum_{i=1}^{n} w_i P(x_i, L_i)} + 1 \\
&= \frac{E[x_p] - L_p}{e^{r} \sum_{i=1}^{n} w_i P(x_i, L_i)} + 1,
\end{align*}
\]

in which the last equation is guaranteed by (5). The constants \(e^{r}\) and 1 can be ignored from the last expression since they do not play any role when the expression is used as a performance measure. By this transformation, the portfolio Omega is equivalent to the ratio of the excess return of the portfolio over the portfolio loss threshold to the weighted average price of the puts after the constants are discarded. That means the portfolio Omega can be alternatively defined as

\[
\Omega_p(w_1, w_2, \ldots, w_n) = \frac{E[x_p] - L_p}{\sum_{i=1}^{n} w_i P(x_i, L_i)},
\]

(7)

This verifies that maximizing portfolio Omega is the same as maximizing an excess return of the portfolio for a given weighted average price of puts. The weighted average price of puts represents a downside risk of the portfolio because the replicated portfolio may lose money due to its short put options when some assets drop values.

**CHARACTERISTICS**

The portfolio Omega has many superb characteristics: It is derived with fewer and realistic assumptions, it is easily used in a portfolio optimization, it can be decomposed by investment strategies or a sector classification, it is an extension to Omega given by Keating and Shadwick (2002b) and the Sharpe-Omega defined by Kazemi et. al, (2004), and portfolio Omega allows investors to put their views into an optimization, and to set their funding cost and investment
strategies in the measure. The drawback of portfolio Omega is that it does not consider correlations among assets. However, it can be addressed in a practice.

First, portfolio Omega has fewer and realistic assumptions. We derive the portfolio Omega under the assumptions that an investor holds every asset in a portfolio over one period, and wants to gain the portfolio value and to avoid a portfolio loss at a certain threshold. The derivation uses only put-call parity without any requirement about return distributions of assets in the portfolio.

An investor can easily set up and solve a portfolio optimization with the portfolio Omega. For a simple setup, an investor can place the weighted average price of call options as an objective function in the optimization while placing a portfolio loss threshold and the downside risk (or the weighted average price of puts) into constraints. Mathematically, the optimization looks like

$$\text{Maximize} : \sum_{i=1}^{n} w_i C(x_i, L_i)$$

subject to:

$$\begin{align*}
\sum_{i=1}^{n} w_i &= 1 \\
\sum_{i=1}^{n} w_i L_i &= L_{\text{loss}} \\
\sum_{i=1}^{n} w_i P(x_i, L_i) &\leq P_{\text{put}},
\end{align*}$$

where $L_{\text{loss}}$ is a targeted loss threshold for the portfolio and $P_{\text{put}}$ is an upper bound for the downside risk. Since the optimization objective and left sides of constraints in (8) are linear functions in assets’ weights, the investor can use a linear programming to solve the optimization without any difficulty.

Please note that we did not include other constraints in the optimization. In practice, an investor can add more constraints to the optimization such as a position limit for each asset. On the other hand, the investor can change the objective function to volatility or a tracking error of the portfolio, and put the weighted average price of call options into the optimization as a constraint. He can solve the optimization using a quadratic programming.

An investor can decompose portfolio Omega according to investment strategies or a sector classification. A decomposition of performance measure is crucial in portfolio management since it is able to provide detailed information about performances associated with investment strategies or a portfolio composition. A good decomposition can aid investors to make better investment decisions and to improve their performance. Unfortunately, most of performance measures do not have decomposition. Portfolio Omega is exceptional and does have decomposition. An investor can decompose portfolio Omega in this way. Suppose each asset in (6) represent an investment strategy or a sector. Portfolio Omega in (6) can be transformed to

$$\Omega_p(w_1, w_2, \ldots, w_n) = \frac{\sum_{i=1}^{n} w_i P(x_i, L_i) \Omega_i(w_i)}{\sum_{i=1}^{n} w_i P(x_i, L_i)} = \sum_{i=1}^{n} W_i \Omega_i(w_i),$$

where
$$\Omega_i(w_i) = \frac{C(x_i, L_i)}{P(x_i, L_i)}$$

is Omega corresponding to strategy or sector $i$, and

$$W_i = \frac{w_i P(x_i, L_i)}{\sum_{i=1}^{n} w_i P(x_i, L_i)}$$

is the percentage contribution of strategy or sector $i$ to the portfolio downside risk (while the term $w_i P(x_i, L_i)$ is the contribution of strategy or sector $i$ to the portfolio downside risk), $i = 1, 2, \cdots, n$. The summation of all percentage contributions (or all contributions) is one (or the portfolio downside risk). Therefore, portfolio Omega in (9) is in fact a downside risk-weighted average of the Omegas, each of which is the one linked to each investment strategy or each sector.

Portfolio Omega is an extension to the original Omega given by Keating and Shadwick (2002b), and the Sharpe-Omega proposed by Kazemi et. al (2004). The Sharpe-Omega is defined as a ratio of an excess return of an asset over a loss threshold to the price of a put option on the asset. Indeed, when a portfolio has only one asset, portfolio Omega becomes

$$\Omega_p(w_i) = \frac{C(x_i, L_i)}{P(x_i, L_i)} = \Omega(L_i)$$

by (6), which is the original Omega expressed in (2), and

$$\Omega_p(w_i) = \frac{E[x_i] - L_i}{P(x_i, L_i)}$$

by (7), which is the Sharpe-Omega.

An investor is allowed to incorporate his views in an optimization of portfolio Omega. He can directly change prices of call and put options in portfolio Omega based on his estimation of assets’ distributions or options’ prices. He can forecast prices of the options using prior distributions of assets in the portfolio updated by posterior information, or he can integrate outputs from an alpha model into the optimization if such model is available to him. In addition, the investor can use the decomposition of portfolio Omega to do an optimization at each strategy or sector, and then do the optimization among the strategies or sectors at a portfolio level. The investor can classify strategies or sectors with his views on markets.

At last, an investor is able to align his financing cost with a portfolio loss threshold. One example is that a money manager receives funding from a financial source and invests in a portfolio. He faces a funding cost to hold the portfolio. He may set a portfolio loss threshold to the cost plus a cushion. The loss threshold may be a targeted excess return of the portfolio over a benchmark for a traditional money manager and a funding rate plus a spread for a hedge fund manager. In another example, a money manager finances his purchase of assets by short selling other assets in
the portfolio. The manager may set a loss threshold as a targeted return spread between the purchased assets and the short sold ones. Furthermore, an investor can line up his investment strategy with a loss threshold of an asset. For instance, a loss threshold can be the total cost in a directional bet with the asset or it can be a targeted relative value in a pair trade involving the asset.

Even though portfolio Omega has many excellent characteristics, it does have the drawback that it does not incorporate correlations among assets in a portfolio. This drawback can be seen by a simple illustration. Suppose there are two portfolios, each of which has two assets with the same weight, and the options on the assets in the first portfolio and the ones on the assets in the second portfolio have the same prices. The two portfolios will have the same portfolio Omega based on its definition regardless of a correlation between the two assets in either of portfolios. To deal with the drawback of portfolio Omega, we suggest using portfolio Omega in diversified portfolios. For example, many portfolios have benchmark constraints that set position limits or maximum allowable under-weights. In these contexts, portfolio guidelines compensate for the drawback in portfolio Omega. On the other hand, if an investor uses portfolio Omega in an optimization, he can introduce some quadratic risk measure such as volatility or tracking error as an objective function or a constraint in the optimization.

EMPirical WORK

In this section, we first analyze an impact of a portfolio loss threshold on portfolio Omega. Then we investigate relationships among portfolio Omega, diversification, loss threshold, and downside risk of a portfolio through an empirical work. Both analysis and empirical work show the same conclusion: portfolio Omega is consistent with practices used in the industry.

A choice of a portfolio loss threshold has a big impact on portfolio Omega although the threshold does not appear in the definition of Omega in (6). When an investor can tolerate a high loss threshold in a portfolio, more weights will be assigned to the call and put options with high strike prices in the replicated portfolio. The investor has a small chance of exercising the call options with high strike prices while the put options with high strike prices have a big chance of being exercised by his counterparties. Therefore, the investor may not make money with the call options and may lose money with the put options. In this case, the portfolio Omega becomes smaller. Alternatively, if the investor sets a small loss threshold, the call and put options with low strike prices will be assigned to more weights in the replicated portfolio. The investor may make money since the call options likely are in-the-money and the put options likely are out-of-money. As a result, the portfolio Omega becomes bigger. These analyses indicate that an investor who can tolerate a large loss has a smaller portfolio Omega than a loss-averse investor. This is also verified by a later empirical work.

Next, we investigate relationships among portfolio Omega, diversification, a loss threshold, and downside risk of a portfolio through an empirical work. We carry out the empirical work using the optimization in (8). In order to clearly observe the relationships, we do not include other constraints in the optimization. The empirical work is set up as follows.

---

5 It is assumed that there are no transaction costs, no margin requirements, and no restrictions on short selling.
Suppose that a portfolio consist of 30 stocks from the Dow Jones Industrial Average. The portfolio holds each stock in the portfolio over a 30-day period. There is no dividend payment from each stock during the period. The European call and put options written on the stocks are priced with the Black-Scholes formula. The volatilities in the formula are at-the-money 30-day implied volatilities of the stocks for all strike prices\(^6\). They are dated on June 25\(^{th}\), 2008. Note that the calculated prices of the options may be different from the ones under the true measure, but we have no way of knowing the later prices.

In the optimization in (8), the targeted portfolio loss threshold is \(L_{\text{loss}}\) and the upper bound for a downside risk is \(P_{\text{put}}\). The loss threshold for each stock is the same as the one for the portfolio. For each value of \(L_{\text{loss}}\) and each value of \(P_{\text{put}}\), a solution to the optimization can be found through the MatLab function \(fmincon\).

We graph solutions to the optimization in the Figures 1 - 4. Figure 1 describes the relationship between portfolio Omega and a loss threshold for a given downside risk. As we analyzed in the beginning of this section, portfolio Omega becomes smaller when a loss threshold increases. It implies that portfolio Omega is a smaller performance measure for an investor who can bear a big loss in the portfolio and it is a larger performance measure for a loss-averse investor. Portfolio Omega is the same for all downside risks if the investor is loss-neutral, i.e., his loss threshold is zero.

Figure 2 shows the relationship between portfolio Omega and a downside risk of the portfolio for a given portfolio loss threshold. For a low loss threshold, portfolio Omega becomes smaller as the downside risk increases in the portfolio. It means an investor with loss aversion cannot gain portfolio Omega by taking more downside risk. On the other hand, for a high loss threshold, portfolio Omega becomes larger as the downside risk increases in the portfolio. This shows that an investor with a high loss threshold can take a more downside risk to achieve a high portfolio Omega. In addition, the curve in Figure 1 is quite flat for a loss neutral investor in spite of a size of downside risk in the portfolio.

Figures 3 and 4 illustrate the relationship between portfolio diversification and a loss threshold for a given downside risk or between portfolio diversification and a downside risk of the portfolio for a given loss threshold. The diversification of the portfolio is approximately measured as the standard deviation of weights of the 30 stocks\(^7\). Figure 3 shows that the portfolio is more diversified when a loss threshold is smaller and more concentrated when a loss threshold is larger for all sizes of downside risk. This may explain why an investor with loss aversion may have a more diversified portfolio than an investor who can stand a loss. Figure 4 shows that the portfolio is more diversified when a downside risk is lower and less diversified when a downside risk is higher for all loss thresholds. This is consistent with the fact that a portfolio with a low risk should be well-diversified or vice versa. Since the results in the Figures 3 and 4 are based on the optimization setup, any conclusion on these results may be not held in a general case.

---

\(^6\) We should use different implied volatilities associated with different strike prices in the Black-Scholes formula. However, the strike prices used in this example are very close to market stock prices on June 25\(^{th}\), 2008. The error due to the misuse of volatilities is small.

\(^7\) A high standard deviation implies a few of stocks have more concentrated weights. In this case, the diversification of portfolio is low. Conversely, a low standard derivation means more weights are close to each other. So an equally-weighted portfolio is the most diversified among all portfolios according to this definition of diversification.
CONCLUSION

We proposed portfolio Omega in order to use the concept of Omega in a portfolio optimization. We started with replicating a portfolio with a group of call options, a group of put options, and cash. The weighted average price of call options represents an upper potential in the replicated portfolio while the weighted average price of put options stands for the downside risk of the portfolio. The cash corresponds to a portfolio loss threshold, which expresses how an investor’s attitude towards a loss in a portfolio. We argued that an investor likes the ratio of the weighted average price of call options to the downside risk at a given a portfolio loss threshold if the investor wishes to gain his wealth, and to avoid a loss at the threshold. We defined the ratio as portfolio Omega.

We discussed and explained the characteristics of portfolio Omega. We demonstrated how to set up a portfolio optimization with portfolio Omega and to solve the optimization. We showed a decomposition of portfolio Omega by strategies or sectors. We pointed up that portfolio Omega is in fact an extension of Omega (Keating and Shadwick, 2002b) and the Sharpe-Omega (Kazemi et. al, 2004). We illustrated how to put views on markets into an optimization and to set funding costs or investment strategies in the measure. We gave an example to illustrate the drawback of portfolio Omega, which does consider correlation among assets in a portfolio, and provided possible solutions to address the drawback.

We analyzed relationships among portfolio Omega, diversification, loss threshold, and downside risk of a portfolio. The portfolio consists of 30 stocks in the Dow Jones Industrial Average. The analyses point to that an investor with loss-aversion has a higher portfolio Omega than the one who can bear a big loss. An investor with loss-aversion cannot gain his portfolio Omega by taking more downside risk while an investor, who can tolerate a big loss, does increase his portfolio Omega by taking more downside risk. A portfolio with a low loss threshold is more diversified than the one with a high loss threshold and a portfolio with less downside risk is more diversified than the one with more downside risk. The analyses reveal portfolio Omega is consistent with practices used in the industry.

ACKNOWLEDGEMENTS

We would like to thank Dr. Xuesong Hu for providing his critical comments and excellent example. We appreciate his promptness helping us get charts in the example. We also thank Dr. Benjamin Tarlow for his recommendation on the structure of paper.
REFERENCES


FIGURES

Figure 1: The portfolio Omega of the portfolio (30 stocks from Dow Jones Industrial Average) versus a portfolio loss threshold (weighted average loss threshold of all stocks, from -2% to 2%) for different given downside risks (upper bound for the weighted average price of put options, from 4% to 8%)

Figure 2: The portfolio Omega of the portfolio (30 stocks from Dow Jones Industrial Average) versus a downside risk (upper bound for the weighted average price of put options, from 4% to 8%) for different given portfolio loss thresholds (weighted average loss threshold from -2% to 2%)
Figure 3: Diversification of the portfolio (30 stocks from Dow Jones Industrial Average) versus a portfolio loss threshold (weighted average loss threshold of all stocks, from -2% to 2%) for different given downside risks (upper bound for the weighted average price of put options, from 4% to 8%). The diversification is defined as the standard deviation of the weights in the portfolio.

Figure 4: Diversification of the portfolio (30 stocks from Dow Jones Industrial Average) versus a downside risk (upper bound for the weighted average price of put options, from 4% to 8%) for different given portfolio loss thresholds (weighted average loss threshold from -2% to 2%). The diversification is defined as the standard deviation of the weights in the portfolio.